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DIFFERENT PROBLEM CLASSES AND SOLUTION TECHNIQUES FOR MODEL PREDICTIVE BUILDING CONTROL

Ján Drgoňa[a,∗], Lieve Helsen[a,b]

[a] *KU Leuven, Department of Mechanical Engineering, Leuven, Belgium*
[b] *EnergyVille, Thor Park, Waterschei, Belgium*

Abstract

It has been proven that the building sector can significantly benefit from replacing current practice rule-based controllers (RBC) and adopting advanced control strategies like model predictive control (MPC). Despite this fact, the application of MPC in practice is still in its early stages. First reason behind this is that MPC requires an accurate and computationally tractable model to predict and optimize the future behavior of the building by adjusting the control actions correspondingly. Second, the solution of the corresponding optimal control problem imposes increased hardware and software requirements in terms of computational power and dedicated software tools. To effectively overcome these barriers in practice, it is, therefore, necessary to take them into account already in the early stages of the MPC design and deliver a customized solution for each particular case. This paper provides an overview of most notable MPC problem classes and corresponding solution techniques suitable for building climate control applications. The conceptual comparison of selected solution approaches summarizes their general characteristics, available solvers and their advantages and disadvantages. Furthermore, for practical reasons the paper provides a list of dedicated optimization solvers which can be used to solve MPC problems in the context of building climate control.
This paper fits in the special hybridGEOTABS track.

Keywords: Model predictive control, Building climate control, Energy performance of buildings, MPC formulation, MPC problem classes

1. Introduction

The energy used by buildings in developed countries accounts roughly for 40% of the overall energy use, most of which is being used for heating, cooling, ventilation, and air-conditioning (HVAC) [1]. Energy savings thus become a priority in the design and operation of the modern HVAC systems. It has been proven that advanced HVAC control can notably reduce the energy use and mitigate emissions of greenhouse gases [2]. Based on this potential, recently revised EU policy on the energy performance of buildings states that large buildings should be equipped with building automation and control systems by 2025 if economically and technically feasible [3]. Nevertheless, the majority of the buildings still adopt simple rule-based control techniques with only limited energy saving capabilities [4]. However, the decrease in the cost of computation and sensing in recent years paved the way for the adoption of advanced control strategies widely used in industry for decades. Above all research on the use of model predictive control (MPC) for buildings has intensified in the last decade [5].
The purpose of this paper is to provide a compact yet comprehensive higher level overview of the different MPC formulations and solution techniques suitable for building control application. The paper is organized as follows. The basics of MPC and its building control aspects are explained in Section 2. Different MPC problem classes and solution approaches are briefly summarized in Sections 3 and 4, respectively. While

∗Corresponding author
Email addresses: jan.drgona@kuleuven.be (Ján Drgoňa), lieve.helsen@kuleuven.be (Lieve Helsen)

Section 5 provides a comprehensive list of available software tools for modeling, analysis and solution of MPC problems. Finally the conclusions are drawn in Section 6.

2. Model Predictive Control for Buildings

MPC is an optimal control strategy that calculates the optimal control inputs by minimizing a given objective function over a chosen prediction horizon, using the mathematical model of the system and current state measurements to predict the future behavior, while taking these future predictions, disturbances and additional constraints into account during the optimization.

2.1. General MPC Formulation

The general MPC formulation can be cast as an optimal control problem (OCP) in discrete time as presented by Equation (1).

$$
\begin{aligned}
\min_{u_0,\dots,u_{N-1}} \; & \ell_N(x_N) + \sum_{k=0}^{N-1} \ell(x_k, u_k) && && \text{(1a)} \\
\text{s.t. } & x_{k+1} = f(x_k, u_k, d_k), && k \in \mathbb{N}_0^{N-1} && \text{(1b)} \\
& x_k \in \mathcal{X}, && k \in \mathbb{N}_0^{N-1} && \text{(1c)} \\
& u_k \in \mathcal{U}, && k \in \mathbb{N}_0^{N-1} && \text{(1d)} \\
& x_0 = x(t), && && \text{(1e)} \\
& d_0 = d(t), && && \text{(1f)}
\end{aligned}
$$

Where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^m$ and $d_k \in \mathbb{R}^q$ denote the values of states, inputs and disturbances, respectively, at the k-th step of the prediction horizon N. The objective function is given by (1a) where $\ell_N(x_N)$ represents the terminal penalty, while $\ell(x_k, u_k)$ is called a stage cost and its purpose is to assign a cost to a particular choice of x_k and u_k. The predictions are obtained from the controller model (1b), and states and inputs are subject to constraints (1c) and (1d). For generality we can denote by ξ the vector that encapsulates all time-varying parameters of (1), i.e. the current states $x(t)$, current and future disturbances $d(t), \dots, d(t+NT_s)$, and possibly other parameters such as comfort boundaries or reference signals.

2.2. Objectives in Building Control

The objective, or also called cost function, represents the performance target of the controlled building, which is to maximize thermal comfort and minimize energy use. However, other objectives, such as maximizing the share of renewables, minimizing CO_2 emissions, etc., can be also defined. Thermal comfort is usually achieved by keeping the zone temperatures of the building in a given comfort range as defined by the European standard ISO-7730. A more advanced and commonly used comfort index is the Predicted Mean Vote (PMV) [6]. PMV, however, is nonlinear and complicated to calculate such that it fits real observed mean vote in practice [7]. It should be noted that comfort and energy use are competing objectives and thus weighting factors need to be introduced to balance the objective and set up the balanced trade-off between both. On top of that important setup and tuning factors of the MPC are: prediction horizon N and sampling period T_s, which are usually determined by the dynamics of the controlled system. The prediction horizon defines the length of a time window for which MPC computes the predictions given by model (1b) and enforces the desired system behavior by the objective function. The sampling time is the time interval in which the computed control actions remain constant. Buildings are in principle slow dynamic systems with T_s usually spanning from 15 to 180 minutes. N for building control applications usually spans between 5 to 48 hours. More detailed overviews, comparisons, and strategies for selection of an appropriate objective function for MPC-based building control can be found e.g. in [8].

2.3. Building Models for Control

The main bottleneck for practical implementations of MPC is to obtain an accurate controller model (1b) of the whole building without the need for engineering expertise, at a low cost, and simple enough to be usable in optimization [9]. Detailed building energy simulation softwares (BES) such as EnergyPlus, TRNSYS or Modelica can provide accurate building models, which are however in general too complex to be used in efficient optimization algorithms. Hence, simplifications are needed to increase the computational tractability [10]. There are three main approaches how to obtain such controller models. Data driven *grey-box* [11] and *black-box* [12], or simplified first principle *white-box* modeling [13]. Reviews and comparison of different modeling techniques for HVAC systems can be found in [14].

2.4. Constraints Used in Building Control

MPC can handle a wide variety of constraints on state, input or output variables [15]. In general, there are two types of constraints, inequality (control inputs range, comfort zones, etc.) and equality (building model dynamics, rate limits, etc.) constraints. The constraints for which satisfaction is mandatory are called *hard constraints*. The constraints which can be violated are known as *soft constraints*, these are typically relaxed by additional slack variables that are penalized in the objective function (1a). Soft constraints are preferable for numerical reasons to guarantee the feasibility of the resulting optimization problem at every condition. Another type of constraints from a practical point of view are time-varying constraints, slew rate constraints which penalize the rate of change of certain variables, or terminal constraints which can be used for enforcing the stability and recursive feasibility. Most commonly in building control applications, the constraints are used to enforce selected variables to stay within given ranges, e.g., heat influxes and room temperatures [16], supply air temperature [17], air flow rate [18] and many more [8]. From mathematical point of view, the constraints can be further linear or nonlinear or incorporate integer variables, which further results in increased complexity of the resulting optimization problem as will be discussed in Section 3.

2.5. Implementation of MPC via Receding Horizon Control

Conventionally MPC algorithms are being implemented in closed-loop configuration using the principle of *receding horizon control* (RHC), where the prediction horizon keeps being shifted forward, implementing only the first step of the computed control strategy and discarding the rest. In general, there are three basic methods to solve the OCP (1): first, methods based on Hamilton-Jacobi-Bellman (HJB) equations and *dynamic programming*; second *indirect methods* based on the calculus of variations and Pontryagin's maximum (minimum) principle; and third, *direct methods* based on the translation of OCP (1) to the corresponding optimization problem (OP) and solution via optimization algorithms, which is mostly used nowadays [19].

3. MPC Problem Classes

In this section we recall the most notable MPC problem classes which differ in the type of corresponding optimization problem to be solved via *direct methods*.

3.1. Linear MPC

We speak about linear MPC (L-MPC), when the objective function (1a) is either linear or quadratic and the prediction model (1b) is linear

$$x_{k+1} = Ax_k + Bu_k + Ed_k, \tag{2a}$$
$$y_k = Cx_k + Du_k, \tag{2b}$$

where y_k denotes the output value at the k-th step. Then OCP (1) can be translated into a Linear Programming (LP) or Quadratic Programming (QP) problem, respectively. There are two methods for this translation, *sparse* or *dense* approach [20]. Due to the fact that building envelope thermal dynamics can be linearized with high accuracy [21], the L-MPC is considered to be a mature technique in the building climate control sector [17].

3.2. Nonlinear MPC

Nonlinear MPC (N-MPC) emerges, when either the objective function (1a) or the prediction model (1b) are nonlinear. Then the translation of OCP (1) yields a Nonlinear Programming (NLP) problem. Tthre particular methods are used for the translation nowadays [19], namely *single shooting*, *multiple shooting* and *collocation* method. In all three cases, the resulting NLP can be efficiently solved by using, e.g., sequential quadratic programming (SQP) algorithms [22], even on larger scales. N-MPC has a big potential in the building sector due to the potentially more accurate predictions of the nonlinear models (HVAC models in particular) and higher flexibility in the formulation of OCP (1). Up to the date, several studies and real applications of N-MPC for buildings have already been reported [23], and we can expect more to come in the following years.

3.3. Hybrid MPC

When the dynamic system model (1b) employs switching dynamics, binary or integer control variables, logic states or constraints then we speak about hybrid MPC (H-MPC). On the one hand, if the hybrid dynamic model is piecewise linear the corresponding optimization problem to be solved is either a Mixed-Integer Linear Programming (MILP) or Mixed-Integer Quadratic Programming (MIQP) problem, w.r.t. the type of the objective function. On the other hand, if the hybrid dynamic model incorporates nonlinearities we end up with an extremely difficult Mixed-Integer Nonlinear Programming (MINLP) problem. Two main frameworks exist for modeling hybrid systems for control purposes: *Mixed logical dynamical* models [24] or so-called *Big-M* modeling approach [25]. H-MPC is a powerful tool for control of buildings employing discrete decision variables (e.g., shadings positions, on-off valves, etc.) [26] or switching dynamics (e.g., operating modes of the heat pump) [27].

4. Direct MPC Problem Solutions

In this section, we recall three distinct solution paradigms based on *direct methods* which can be used to obtain solutions to the MPC problems described in the previous section.

4.1. Implicit MPC

In the case of implicit MPC, the optimal control sequence U_N^* for a particular initial condition ξ is obtained by solving the corresponding optimization problem obtained by recasting OCP (1). The computational complexity associated with obtaining such a sequence depends on the type of the prediction model (1b) and the choice of the cost function (1a), as discussed in Section 3. An overview of the most notable optimization solvers for each class of problems is provided in Section 5. As mentioned in Section 2.2 buildings are inherently slow dynamic systems which allow sufficiently large time windows for the solution of large-scale OP emerging from MPC problems with longer prediction horizons and a larger number of parameters, which are typical for building control applications. Hence there is no surprise that most of the building MPC applications reported in a survey [8] have been implemented online via implicit MPC.

4.2. Explicit MPC

The basic idea in explicit MPC is to employ *parametric programming* [28] to pre-calculate the optimal control inputs for *all* admissible values of parameters. Hence the explicit representation of the optimizer is constructed off-line as a function of the vector of initial conditions (1e), (1f). Then, the on-line identification of the optimal control action boils down to mere function evaluations for particular measurements. This significantly reduces computational requirements of the implementation. From a mathematical point of view, the problems to be solved in the case of linear MPC are multi-parametric linear programs (mpLP) or multi-parametric quadratic programs (mpQP), respectively. The fundamental limitation of the explicit MPC approach is, however, that the complexity of the computed explicit control law grows exponentially with the dimensionality of the parametric space. Therefore it can only be used for small-scale systems with up to 10 parameters [29]. These restrictions are usually not a realistic assumption for complex building control problems with several thousands of parameters and hundreds of optimization variables. Thus only a few applications of explicit MPC for building control have been reported [30].

4.3. Approximate Explicit MPC via Machine Learning

The basic idea in approximate explicit MPC is to use machine learning (ML) algorithms for learning simplified control laws from the MPC teacher with an arbitrary type of cost function and constraints. Regression algorithms are used for problems with continuous control variables and classification for problems with discrete control variables. The advantage over implicit MPC is that the solution of the optimization is replaced by computationally cheap function evaluations similarly as in the case of explicit MPC. The main advantage over the explicit MPC approach, is that the ML approach is not limited to lower dimensional parametric spaces, which allows construction of the approximated explicit control laws with low memory footprint also for large-scale problems with many parameters. The drawback of the ML approach is that control policy is suboptimal w.r.t. the solution of the MPC problem (1). The promise of delivering low complexity solutions to the MPC problems which could be easily implemented even on lower level hardware naturally draws the attention of the building control community. One of the first attempts to generate MPC laws in the forms of look-up tables was introduced by Coffey [31]. Other researchers used classification algorithms [32] for extracting control rules from MPC employing integer decision variables, and regression algorithms [33] for MPC with continuous decision variables.

5. MPC Software Tools

Nowadays dozens of optimization solvers are available, both commercial and free for a wide variety of problems. Tab. 1 and Tab. 2 provide an overview of the most important solvers suitable to solve MPC problems on a desktop and embedded platforms, respectively. Progress in the solution techniques and increase in the computational power of the desktop platforms allows us to efficiently solve large-scale optimization problems with up to hundreds of thousands of variables on-line in an implicit way. In the case of embedded platforms, several tools come up with automated and optimized code generation features supporting different languages (e.g., C, C++ or Python) for rapid development and deployment of MPC for real-world applications. These embedded applications are however suitable for small but fast dynamic systems which are on the other side of the track when compared to the large and slow dynamics of the buildings. Nevertheless their efficiency and cheap computational power could be harnessed in smaller scale residential applications of MPC.

Table 1: Overview of the most notable optimization solvers suitable to solve MPC problems on desktop platforms.

Solver	**Free**	**LP**	**QP**	**MILP**	**MIQP**	**MINLP**	**NLP**
CPLEX	-	•	•	•	•	-	-
Gurobi	-	•	•	•	•	-	-
MOSEK	-	•	•	•	•	-	-
XPRESS	-	•	•	•	•	-	-
SeDuMi	•	•	•	-	-	-	-
SDPT3	•	•	•	-	-	-	-
CVXOPT	•	•	•	-	-	-	-
GLPK	•	•	-	•	-	-	-
IPOPT	•	•	•	-	-	-	•
KINTRO	-	•	•	-	-	-	•
SNOPT	-	•	•	-	-	-	•
APOPT	-	•	•	•	•	•	•
BARON	-	•	•	•	•	•	•

Table 2: Overview of the most notable optimization software tools suitable to solve MPC problems on embedded platforms.

Solver	**Free**	**Code Generation**	**LP**	**QP**	**mpLP/ mpQP**	**MILP/ MIQP**	**NLP**
OOQP	•	-	•	•	-	-	-
qpOASES	•	-	•	•	-	-	-
ECOS	•	-	•	•	-	-	-
CVXGEN	•	•	•	•	-	-	-
FiOrdOs	•	•	•	•	-	-	-
FORCES PRO	-	•	•	•	-	-	-
Falcopt	•	•	•	•	-	-	•
Toolbox							
ACADO	•	•	•	•	-	-	•
MPT3	•	•	•	•	•	•	-
Hybrid Toolbox	•	•	•	•	•	•	-
MPC Toolbox^{TM}	-	•	•	•	•	•	•

In the case of data-driven approximate MPC, the machine learning models can be trained by solving a wide variety of optimization problems off-line. The type of the OP to be solved depends on the used models (e.g., neural networks, regression trees, etc.) and their specification. While also dedicated algorithms exist to train more complex and specific ML models [34], most of the problems can be solved by using general purpose solvers listed in Tab 1.

6. Conclusions

This paper provides an overview of the MPC specifications for building control applications (building model, type of constraints and objective functions), its most notable problem classes (linear, nonlinear, hybrid), solution techniques (implicit, explicit, approximate), and summarizes dedicated software tools for solution and deployment of MPC controllers.

The details of the particular case, such as building model type (e.g., linear, nonlinear), comfort index (e.g., comfort range, PMV), and imposed constraints determine the formulation of the resulting MPC problem. The MPC formulation then defines the type of optimization problem with its computational complexity and thus determines the feasibility, maximum size of the problem to be solved as well as the hardware and software requirements for each particular solution approach. Linear MPC formulation is computationally least demanding and thus easiest to implement. Many modeling tools support linear MPC with a wide variety of examples and tutorials. Even though it has certain limitations regarding problem formulation it is a very suitable and easy to deploy approach for most of the applications thanks to the fact that building envelope has in principle linear dynamics. Nonlinear MPC provides higher flexibility in formulation, and due to the higher accuracy of the nonlinear HVAC model, it can provide increased performance compared to the linear case. However, this comes with an increased cost of computational demands and the risk that local (instead of global) optima are found. Hybrid MPC is useful when one needs to deal with integer decision variables or switching dynamics like heat pump modes, etc., which are very common in building applications. For the cost of increased computational demands, it can provide increased performance compared to more straightforward linear case. This allows to compute the optimal control signals directly for lower level hardware of the building (such as pumps and valves) instead of heat fluxes as often the case with linear MPC. Due to the specific nature of building climate control applications, such as large size of the optimization problem, large sampling times and availability of computational power, MPC is most often being implemented in an implicit way, solving a corresponding optimization problem online. Explicit MPC has been proven to be feasible so far only for smaller single zone case studies. Approximate explicit MPC appears to be a promising alternative providing complex MPC-like controllers which are readily implementable even on lower level hardware. However thi approach requires larger datasets for training, which can be difficult and time-consuming to generate and an original MPC controller design is still necessary to act as a teacher. For all types of MPC formulations and implementation approaches a wide variety of solvers are available nowadays paving the way for many more successful MPC implementations for building climate control problems in the near future.

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